isobaric, while gas motion under the action of a pressure head may be considered isothermal. Without performing calculations, this conclusion is not obvious for a rarefied gas.

NOTATION

ℓ, channel length; a, channel height; P, pressure; T, temperature; f, distribution function; m, molecular mass; k, Boltzmann's constant; v, molecular velocity; x, y, z, coordinates; τ_w, relative wall temperature; c, dimensionless molecular velocity; $L = \ell/a$, dimensionless channel length; η, dynamic gas viscosity; $\mathbf{w}$, dimensionless thermal molecular velocity; u, gas velocity; n, gas density; q, thermal flux density; J_k, thermodynamic flux; Λ_{kn}, kinetic coefficient; X_k, thermodynamic force; h, disturbance function; $L_s h$, collision operator; ϑ, relative gas pressure; τ, relative gas temperature; δ, rarefaction parameter; γ, TPD index.

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ACTION OF THE HALL EFFECT ON FLOW AND HEAT TRANSPORT IN A CONDUCTIVE GAS FLOW NEAR A ROTATING DISK

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The action of the tensor character of medium conductivity upon flow and heat transport in the boundary layer on a rotating disk is studied in the presence of an axial magnetic field and various directions of the angular velocity vector.

In an analysis of an MHD-boundary layer on a rotating disk [1] proposed an approximate method for integrating the nonlinear equations of motions, involving averaging of inertial terms over layer thickness. A modification of that method was later successfully used to calculate hydrodynamic and thermal boundary layers near a rotating disk with exhaust and draft of the medium through the porous surface of the body flowed over in the presence or absence of an external magnetic field [2-5]. Comparison of the moments of the friction forces and thermal fluxes calculated on the basis of the approximate and numerical methods revealed good agreement. In the present study the method of partial consideration of inertial terms will be used to determine flow and heat transport in a flow of conductive gas in the boundary layer on a rotating disk in the case where Hall phenomena play a significant role.

We will consider the motion of a viscous conductive gaseous medium near an infinite dielectric disk rotating at constant angular velocity $\boldsymbol{\omega}$ about the z axis in an external homogeneous axial magnetic field $\mathbf{B}$. Neglecting the induced magnetic field, the system of hydrodynamic equations of the boundary layer with consideration of electromagnetic forces has the form

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$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\frac{\partial^2 u}{\partial z^2}+\frac{j_\varphi B_z}{\rho}, \tag{1}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\frac{\partial^2 v}{\partial z^2}-\frac{j_r B_z}{\rho}, \tag{2}$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0. \tag{3}$$

To determine the values of the densities of the radial j_r and azimuthal j_φ electric currents we use a generalized Ohm's law with neglect of ion drift [6]:

$$j_r=\sigma B_z(v-\beta j_\varphi),\quad j_\varphi=\sigma B_z(\beta j_r-u). \tag{4}$$

Using Eqs. (1)-(4), we obtain the equations of motion in projections on the axes r and φ:

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=\nu\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_z^2 u}{\rho(1+\alpha^2)}+\frac{\sigma B_z^2\alpha v}{\rho(1+\alpha^2)}, \tag{5}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_z^2 v}{\rho(1+\alpha^2)}-\frac{\sigma B_z^2\alpha u}{\rho(1+\alpha^2)}. \tag{6}$$

Here $\alpha = \sigma\beta B_z$ is the Hall parameter.

We note that consideration of Hall effects leads not only to the appearance of additional current components in the radial and tangential directions [the last terms on the right sides of Eqs. (5) and (6)], but to a reduction in the conventional induction current, related to motion of the conductive medium across the magnetic field.

We will assume that $\omega \uparrow\uparrow \mathbf{B}$ and the following approximate boundary conditions are satisfied:

$$\begin{aligned} z=0 \quad & u=0,\quad v=\omega r;\\ z=\delta \quad & u=0,\quad v=0,\quad \frac{\partial v}{\partial z}=0. \end{aligned} \tag{7}$$

Taking $u = rF(z)$, $v = rG(z)$, considering the condition $\partial p/\partial r = 0$, and replacing the inertial terms together with the electromagnetic "forces" by their mean values across the layer section [1], as a result of integrating Eqs. (5), (6) with consideration of Eq. (7) we obtain

$$F=\frac{A\delta_0^2\omega}{2}\left(\frac{\eta^2}{\delta_0^2}-\frac{\eta}{\delta_0}\right),\quad G=\omega\left(1-\frac{\eta}{\delta_0}\right)^2,$$

$$w=\frac{A}{6}\sqrt{\nu\omega}\left(3\frac{\eta^2}{\delta_0^2}-\frac{2\eta^3}{\delta_0^3}\right),$$

where $\eta = z\sqrt{\omega/\nu}$.

The dimensionless quantities A and $\delta_0 = \delta\sqrt{\omega/\nu}$ are defined from a system of algebraic equations:

$$A=\frac{A^2\delta_0^4}{40}-\frac{1}{5}-\frac{S\alpha}{3(1+\alpha^2)}-\frac{SA\delta_0^2}{12(1+\alpha^2)}, \tag{8}$$

$$\frac{2}{\delta_0^4}=A\left[\frac{1}{10}+\frac{S\alpha}{12(1+\alpha^2)}\right]+\frac{S}{3(1+\alpha^2)\delta_0^2}, \tag{9}$$

where $S = \sigma B_z^2/\rho\omega$.

Figure 1 shows the dependence of boundary layer thickness on the parameter α. As $S \to 0$ and $\alpha \to 0$ the solution coincides with the known result for a nonconductive medium [3, 7]. At low values of α the action of Hall effects on the radial electromagnetic force component is insignificant, appearing mainly as a reduction in the braking force [second term on right of Eq. (5)]. For small values of the parameter S this fact leads to an insignificant decrease in boundary layer thickness δ_0. With increase in α the Hall portion of the force

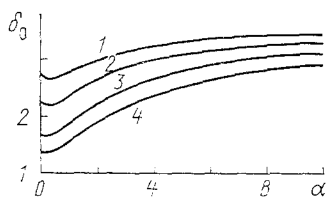

Fig. 1

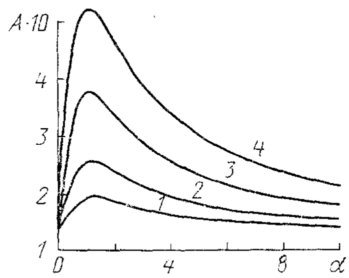

Fig. 2

Fig. 1. Dimensionless boundary layer thickness δ_0 vs Hall parameter α for various values of parameter S: 1) 0.5; 2) 1; 3) 2; 4) 3.16.

Fig. 2. Dimensionless parameter A characterizing intensity of radial flow in boundary layer vs Hall parameter α. S values as in Fig. 1.

plays a more marked role in acceleration of the medium in the radial direction, which causes an increase in δ_0.

The dependence of the dimensionless quantity A on the Hall parameter α is shown in Fig. 2. In contrast to this dependence of A on α, which has a maximum due to proportionality of the electromagnetic force to the quantity $\alpha/(1 + \alpha^2)$, the radial flux in the boundary layer, defined by the expression

$$Q = 2\pi \int_0^\delta v_r r dz = \pi r^2 \sqrt{\nu\omega}\, \frac{A\delta_0^3}{3},$$

increases monotonically with increase in α. This is true because the radial component of the electromagnetic force related to the Hall current accelerates the medium in the direction from the axis to the periphery in this situation. Physically this result can be interpreted as appearance of an "ion wind" upon passage of a radial current across an axial magnetic field under conditions of electron "magnetization" [8].

Analytical solution for the hydrodynamic flow parameters permits easy evaluation of the action of Hall effects on heat transport in the boundary layer with neglect of viscous dissipation and Joulean heating. Using the solution of the thermal conductivity equation [5] for Prandtl numbers $\Pr = \nu/\chi \ll 1$, we obtain an expression for the local Nusselt number:

$$\mathrm{Nu} = \frac{q(0)}{(T_0 - T_1)} (\nu/\omega)^{1/2} = -\frac{A\delta_0^3}{6} \Pr.$$

Here $q(0) = -\varkappa \frac{\partial T}{\partial z}(0)$ is the thermal flux on the disk surface; T_0 and T_1 are the absolute temperatures of the disk and the medium at infinite removal from the disk.

Figure 3 shows the ratio Nu/Pr as a function of the parameter α. As $S \to 0$ ($\alpha \to \infty$) the quantity $\mathrm{Nu}/\Pr \to 0.95$, which agrees well with the data of precise integration ($\mathrm{Nu} = 0.88 \Pr$ [9]). Increase in heat exchange with increase in α is related to increased intensity of secondary flow near the disk surface with growth in Hall effects.

We will note that to analyze the case of oppositely directed magnetic induction $\mathbf{B}$ and disk angular velocity $\boldsymbol{\omega}$ in Eqs. (8) and (9) it is sufficient to replace the parameter α by $-\alpha$. Figure 4 shows results of calculating the radial flux of a conductive gas near the disk surface Q for various directions and values of the external magnetic field. As is evident from the results obtained, in the region $\alpha < 0$ at moderate values of $|\alpha|$ braking of the flow occurs. This can be explained by the change in sign of the Hall portion of the radial electromagnetic force upon change in the direction of the external magnetic field. Increase in intensity of the radial motion of the medium toward the periphery with subsequent increase in $|\alpha|$ is related to decrease in the braking electromagnetic force, proportional to $\alpha/(1 + \alpha^2)$. System (8), (9) has a solution when the azimuthal viscosity forces in the boundary layer compensate the inertial and electromagnetic forces ($1/10 > S\alpha/12(1 +$

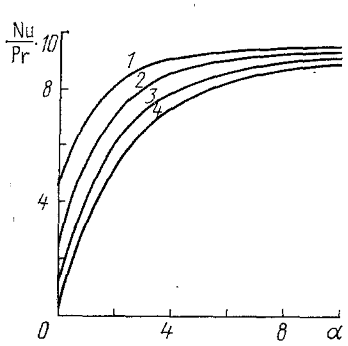

Fig. 3

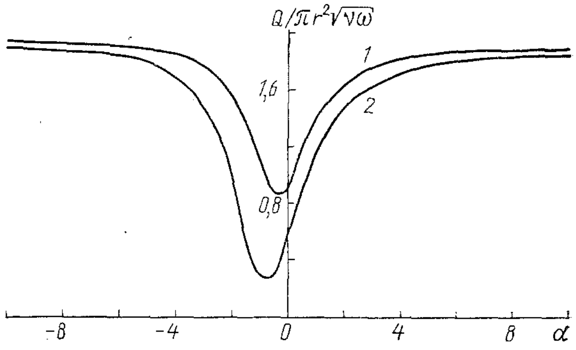

Fig. 4

Fig. 3. Action of Hall parameter α on intensity of local heat exchange on boundary layer at same S values as Figs. 1, 2.

Fig. 4. Radial flux of conductive gas in boundary layer Q vs Hall parameter α for S = 0.5 and 1 (curves 1 and 2, respectively).

α^2)). In the opposite case the type of self-similar motion considered here is not realized in view of the law of conservation of momentum.

The approximate method for calculating electromagnetic forces in the equations of motion of conductive media permits a simple and sufficiently accurate analytical solution of hydrodynamic problems, which may be useful for engineering projections. An advantage of the method is the ease of analysis of various mechanisms which control the behavior of the hydrodynamic characteristics of the boundary layer flow.

NOTATION

u, v, w, radial, azimuthal, and axial velocity components; r, z, radial and axial coordinates; p, pressure; ρ, density; ν, kinematic viscosity; T, absolute temperature; χ, thermal diffusivity coefficient; κ, thermal conductivity coefficient; σ, medium conductivity without magnetic field; $\mathbf{B}$, magnetic field intensity; j, electrical current density; β, Hall constant; ω, angular velocity of disk rotation; δ, disk boundary layer thickness; S, magnetohydrodynamic interaction parameter; Pr, Prandtl number; Nu, Nusselt number; α, Hall parameter; Q, radial flux in boundary layer.

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